Review: Lessons 18-36

April 29, 2016

1. Find the domain of the function $f(x, y) = \dfrac{xe^{1-y}\ln(3x+2)}{(x-y)\sqrt{12x-7y+23}}$.

2. What do the level curves for $g(x, y) = 11\ln\left(2(x+1)^2 + 2(y-3)^2\right)$ look like?

3. Find all first order partial derivatives of $f(x, y) = \frac{x\cos x}{x+2y}$.

4. The productivity of a certain country is given by $Q(K, L) = 120K^{2/3}L^{1/3}$ units, where K is the capital expenditure in units of \$1 million and L is the size of the labor force in thousands of worker-hours. The capital expenditure is currently 4.5 billion dollars and the labor level is 4,215,000 worker-hours. If capital expenditure is to be increased by 4 million dollars, approximately what change should be made to the labor force to increase productivity 500 units?

5. Let $z = e^{x^2+y^2}$, $x = \ln(2t)$, and $y = \frac{1}{t^3-7}$. What is the value of $\frac{dz}{dt}$ when $t = 2$?

6. Find all local maximum and minimum points of $f(x, y) = 3x^2 - xy + y^2 + 44x + 6$.

7. Minimize the cost of a rectangular box with volume 42 cubic feet if the material for the top costs \$6 per square foot, the material for the sides costs \$7 per square foot, and the material for the bottom costs \$8 per square foot.

8. Find the minimum value of the function $f(x, y) = x^2 + y^2$ subject to the constrain $x + y^2 = 18$.

9. The number of foxes and hounds in a certain region are given by $(x-10)^2 + 8(y-14)^2 = 68$, where x is the number of foxes in hundreds and y is the number of hounds in hundreds. What is the maximum number of foxes and hounds in this region?

10. Compute $\int_0^{\pi/4} \int_0^{\sin(x)} \frac{1}{\cos(x)}\, dy\, dx$.

11. Compute $\int_0^1 \int_{2x}^2 \sqrt{y^2 + 3}\, dy\, dx$.

12. Find the average value of the function $f(x, y) = 3e^x\sqrt{y + e^x}$ on the rectangle with vertices $(0,0)$, $(0,1)$, $(5,0)$, and $(5,1)$.

13. Solve the following system of equations:

$$\begin{array}{rcrcrcr} 5x & - & 2y & + & z & = & 20 \\ 2x & - & 4y & + & 3z & = & 4 \\ -6x & + & 2y & + & z & = & 10 \end{array}$$

14. Write the matrix $\begin{bmatrix} 2 & -11 & 5 & 57 \\ -3 & 12 & -6 & -69 \\ 1 & -3 & 2 & 22 \end{bmatrix}$ in reduced row echelon form.

15. Find $A^2 - 3A$ if $A = \begin{bmatrix} 1 & -3 \\ 2 & 7 \end{bmatrix}$.

16. Find the inverse of the matrix $\begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$.

17. Find the determinant of the matrix $\begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}$, and use it to find the inverse.

18. Find the determinant of the matrix $\begin{bmatrix} 1 & -3 & 7 \\ 0 & 1 & 5 \\ -3 & -2 & 3 \end{bmatrix}$.

19. Find the eigenvalues of $\begin{bmatrix} -2 & -10 & 17 \\ -2 & -1 & 2 \\ -3 & -6 & 18 \end{bmatrix}$

20. Which of the following is an eigenvector of $\begin{bmatrix} -2 & -10 & 17 \\ -2 & -1 & 2 \\ -3 & -6 & 18 \end{bmatrix}$?

(a) $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}$

(c) $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$

(d) $\begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}$

Answers:

1. $\{(x, y)|x \neq y, 12x - 7y > -23, x > 2/3\}$

2. Circles with center $(-1, 3)$

3. $f_x = \dfrac{-x^2 \sin x + 2y \cos x - 2xy \sin x}{(x + 2y)^2}, f_y = -\dfrac{2x \cos x}{(x + 2y)^2}$

4. Increase the labor force by 4,470 worker-hours

5. -420.05

6. Local minimum of -170 at $(-8, -4)$

7. \$507.48

8. 17.75

9. 3,300

10. 0.35

11. 2.22

12. 726.13

13. $x = 6.8, y = 16.2, z = 18.4$

14. $\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 8 \end{bmatrix}$

15. $\begin{bmatrix} -8 & -15 \\ 10 & 22 \end{bmatrix}$

16. $\begin{bmatrix} -1/6 & -1/3 & 7/6 \\ -1/3 & 1/3 & 1/3 \\ 1/6 & 1/3 & -1/6 \end{bmatrix}$

17. 2; $\begin{bmatrix} 3 & -1 \\ -1 & 1/2 \end{bmatrix}$

18. 79

19. 5,-3,3

20. a,c